Laser Stripe Profiler Calibration Using Nonlinear Models

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Abstract

The calibration of a laser stripe profiler consisting of a laser stripe projector, camera, and linear motion table is considered. A nonlinear system model is used which accommodates radial distortion in the camera lens. The way in which stripe data is extracted from the camera images leads to a natural formulation of the calibration problem as a nonlinear least squares problem. This can then be solved using standard techniques.

Keywords: *Range sensor, calibration*

1 Introduction

A laser stripe profiler is one of many techniques [4] for generating dense 3D surface information. Such data has uses in computer vision, in computer graphics for generation of realistic object models, and in medical applications.

A laser stripe profiler consists of a laser source, camera, and linear motion platform, in an arrangement such as illustrated in Figure 1. The laser source emits a plane of light which forms an illuminated stripe on the object's surface. This is viewed by a camera displaced from the laser plane. In the indicated arrangement, the stripe runs roughly from top to bottom of the images. The horizontal displacement is related to surface shape. The data extracted from the image is in which column (x-direction) the stripe crosses each row. The position in the x-direction is estimated to subpixel accuracy using a sub-pixel operator such as Gaussian interpolation [6]. Given the camera intrinsic parameters and its relative location with respect to the laser plane, the position of the corresponding point in the laser stripe (i.e., the coordinates in the laser stripe plane) can be estimated by triangulation. Finally, these can be transformed into 3D coordinates by incorporating the known motion of the object relative to the laser plane.

The classic approach to calibration of a laser striping system [1–3,7] is to first use a standard camera calibration technique, such as [9], to estimate the camera parameters. Then the parameters of the laser plane are estimated by projecting observed points into 3D locations and fitting a plane by least squares. This approach has a number of drawbacks. It divides the calibration into two sub-problems which may result in a suboptimal estimate of the parameters. More importantly, it requires an intensity image from the camera. In many systems, this is not feasible because, for example, the camera lens is heavily filtered so that only the laser light is visible, or the images are processed on the fly within the sensor to extract stripe data and thus only stripe position data is available.

The calibration method described in this paper uses a nonlinear camera model and hence requires a nonlinear technique to determine the parameter values. An important contribution of this paper is the formulation of the calibration as a nonlinear least squares problem, which means that standard numerical methods can be used to solve the problem.

The rest of this paper is outlined as follows. In Section 2, the nonlinear system model is described. The details of the calibration procedure are presented in Section 3. The paper is concluded in Section 4 with a discussion of the method's performance and an indication of areas requiring further work.

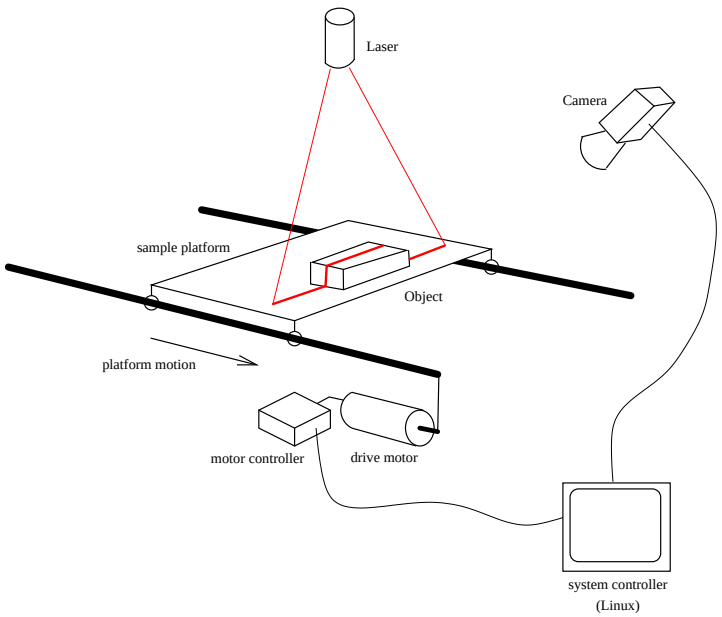

Figure 1: Overview of the laser profiling system.

2 System Description and Model

The laser stripe profiler consists of a camera viewing a scene into which a plane of laser light (generated with a cylindrical lens) is projected. The object of interest is moved through the scene along a linear trajectory. The camera and laser are arranged such that notionally the visible laser stripe appears as a vertical line in the camera's field of view, and such that the direction of motion is notionally along the camera/laser baseline.

Let $\mathbf{x}_w$ be the coordinates of a point on the surface of the object being profiled, in a world coordinate frame fixed relative to the object. The goal of using the system is to recover $\mathbf{x}_w$ for all visible points. Another coordinate frame $\mathbf{x}_f$ is introduced that is fixed with respect to the camera and laser plane. This does not move as the object is moved. The fixed and world frames are defined such that they are co-incident at the start of scanning. The object is moved by a linear translation between images of a constant step size. The relationship between $\mathbf{x}_f$ and $\mathbf{x}_w$ is given by

$$\mathbf{x}_w = \mathbf{x}_f + t\Delta\mathbf{m} \tag{1}$$

where t is the image number (starting at 0), i.e., the "time", Δ is the distance moved between images, and $\mathbf{m}$ is the unit vector representing the direction of travel between images.

A coordinate frame $\mathbf{x}_c$ associated with the camera is introduced. The Euclidian transformation between the fixed and the camera coordinate frames is specified by a rotation $\mathbf{R}$ and a translation $\mathbf{t}$, namely

$$\mathbf{x}_c = \mathbf{R}\mathbf{x}_f + \mathbf{t} \tag{2}$$

The camera model contains a nonlinear term for radial distortion [8]. The transformation from $\mathbf{x}_c$ to $\mathbf{x}_i$ is changed as follows. Let $\mathbf{x}_u$ be undistorted image plane coordinates given by

$$x_u = \frac{x_c}{z_c}, \qquad y_u = \frac{y_c}{z_c} \tag{3}$$

These are related to distorted image plane coordinates $\mathbf{x}_d$ by

$$\mathbf{x}_u = (1 + K_1 r^2)\mathbf{x}_d \tag{4}$$

where $r^2 = x_d^2 + y_d^2$ and K_1 is a radial distortion coefficient. The observed image plane coordinates $\mathbf{x}_i$ are then

$$\mathbf{x}_i = \begin{bmatrix} s_x & k \\ 0 & s_y \end{bmatrix} \mathbf{x}_d + \begin{bmatrix} c_x \\ c_y \end{bmatrix} \tag{5}$$

The light projected by the laser lies in a plane. The plane is specified by a normal $\mathbf{n}$ and an offset d from the origin. In the fixed coordinate frame, the laser plane is thus represented by

$$\mathbf{n}_f^\mathsf{T} \mathbf{x}_f + d_f = 0 \tag{6}$$

Operation of the profiler generates $(\mathbf{x}_i, t)$ tuples for points on the objects surface. From these, an estimate of $\mathbf{x}_w$ is to be generated. From (5),

$$\mathbf{x}_d = \begin{bmatrix} \frac{1}{s_x} & \frac{-k}{s_x s_y} \\ 0 & \frac{1}{s_y} \end{bmatrix} (\mathbf{x}_i - \begin{bmatrix} c_x \\ c_y \end{bmatrix}) \tag{7}$$

Hence $\mathbf{x}_u$ can be calculated from (4), and $\mathbf{x}_c$ expressed as

$$\mathbf{x}_c = z_c \begin{bmatrix} \mathbf{x}_u \\ 1 \end{bmatrix} \tag{8}$$

in terms of the unknown z_c. Substituting this into (2),

$$\mathbf{x}_f = z_c \mathbf{A} - \mathbf{B} \tag{9}$$

where

$$\mathbf{A} = \mathbf{R}^\mathsf{T} \begin{bmatrix} \mathbf{x}_u \\ 1 \end{bmatrix} \tag{10}$$

$$\mathbf{B} = \mathbf{R}^\mathsf{T} \mathbf{t} \tag{11}$$

Substituting this into (6) then gives

$$z_c = \frac{\mathbf{n}_f^\mathsf{T} \mathbf{B} - d_f}{\mathbf{n}_f^\mathsf{T} \mathbf{A}} \tag{12}$$

Thus, $\mathbf{x}_f$ has been determined, and $\mathbf{x}_w$ can be determined from (1).

3 System Calibration

Calibration is the determination of optimal values for the model parameters. This is accomplished by scanning a calibration target about which world information is apriori known. The target used consists of several planar surfaces. The data obtained from scanning the calibration target is presented to an operator as the intensity recorded along the laser stripe for each scan, as shown in Figure 2. The operator identifies the planar regions. This data is used in the parameter estimation process.

In Section 3.1, the calibration process is formulated as a nonlinear regression problem which can be solved using standard tools. The numerical solution technique requires a good starting condition. This is provided by ignoring the nonlinear part of the model and solving the resulting linear system of equations using standard calibration techniques [5].

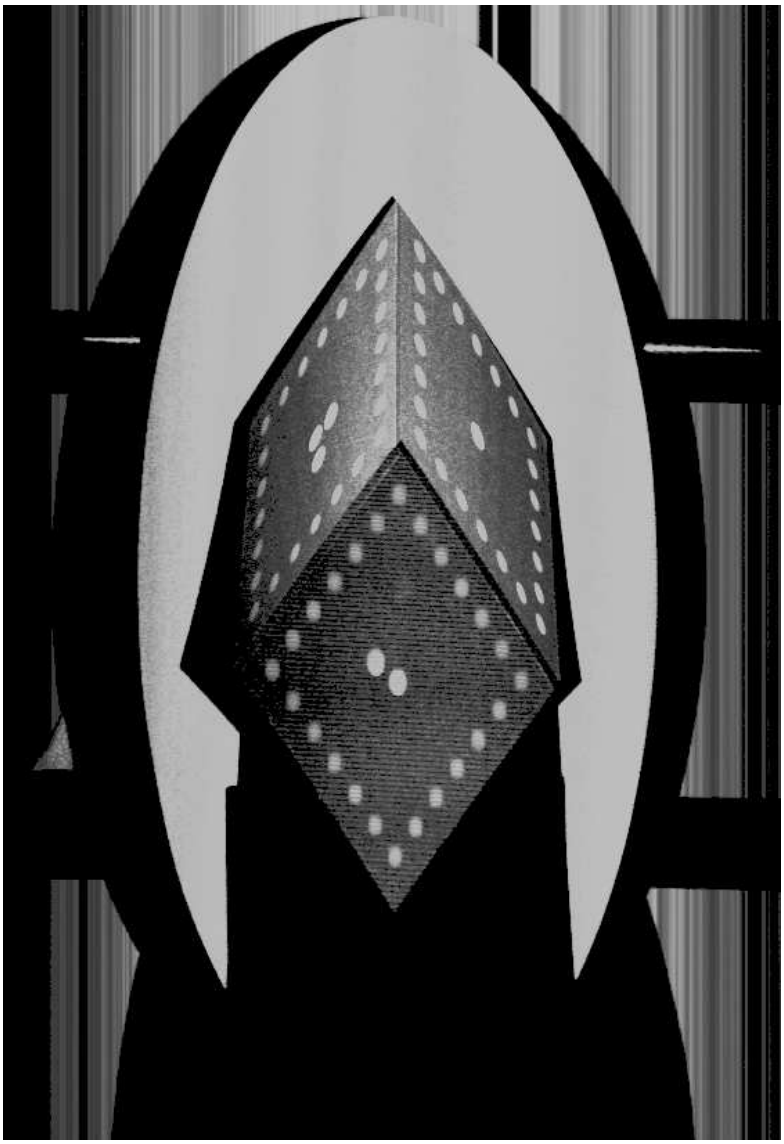

Figure 2: Image constructed from intensity recorded along the laser stripe in each scan. A 3 planar surface target is shown.

3.1 Nonlinear Estimation

The planar surfaces of the calibration target are specified in a world coordinate frame associated with the target as

$$\mathbf{p}_w(k)^{\mathsf{T}}\mathbf{x}_w + q_w(k) = 0, \qquad k = 1 \ldots N \tag{13}$$

The raw data acquired from the target consists of sets of tuples (x_i, y_i, t) for each plane k, which record when the stripe at time t lies on plane k such that the stripe is measured to intersect row y_i at column x_i^o. Thus, if φ represents the system parameters ($\mathbf{R}$, $\mathbf{t}$, s_x, etc.), and the system model can be expressed as

$$x_i = f_k(y_i, t; \varphi) \tag{14}$$

then the system parameters can be estimated by nonlinear least squares minimisation of

$$E = \sum_k \sum_{y_i} (x_i^o - f_k(y_i, t; \varphi))^2 \tag{15}$$

The rest of this section shows how f_k can be derived. The kth calibration target plane (13) can

be expressed in camera coordinates as

$$\mathbf{p}_c^{\mathsf{T}}\mathbf{x}_c + q_c = 0 \tag{16}$$

where the k arguments have been dropped for clarity, and

$$\begin{aligned} \mathbf{p}_c &= \mathbf{R}\mathbf{p}_w(k) & (17)\\ q_c &= q_w(k) + t\Delta\mathbf{p}_w(k)^{\mathsf{T}}\mathbf{m} - \mathbf{p}_c^{\mathsf{T}}\mathbf{t} & (18) \end{aligned}$$

If we define

$$\begin{aligned} \mathbf{n}_c &= \mathbf{R}\mathbf{n}_f & (19)\\ d_c &= d_f - \mathbf{n}_c^{\mathsf{T}}\mathbf{t} & (20) \end{aligned}$$

then (6) can be expressed in the camera coordinate system as

$$\mathbf{n}_c^{\mathsf{T}}\mathbf{x}_c + d_c = 0 \tag{21}$$

and (12) can be rewritten as

$$z_c = \frac{-d_c}{(n_1 x_d + n_2 y_d)(1 + K_1 r^2) + n_3} \tag{22}$$

where the n_i are the components of $\mathbf{n}_c$. Substituting this into (16) gives

$$\begin{aligned} (1 + K_1 r^2)[q_c(n_1 x_d + n_2 y_d) - d_c(p_1 x_d + p_2 y_d)] \\ + q_c n_3 - d_c p_3 = 0 \end{aligned} \tag{23}$$

where the p_i are the components of $\mathbf{p}_c$. Substituting for r, x_d, and y_d from (7) gives a cubic polynomial for x_i in terms of t, y_i, the system parameters, and the calibration target plane parameters:

$$Ax_i^3 + Bx_i^2 + Cx_i + D = 0 \tag{24}$$

where

$$\begin{aligned} A &= K_1\beta^3\alpha_1 & (25)\\ B &= K_1\beta^2(3\alpha_1\gamma + \alpha_2 y_d) & (26)\\ C &= \beta\alpha_1 + K_1\beta(\alpha_1 y_d^2 + 2\alpha_2\gamma y_d + 3\alpha_1\gamma^2) & (27)\\ D &= \alpha_3 + \alpha_1\gamma + \alpha_2 y_d \\ &\quad + K_1(\alpha_2 y_d + \alpha_1\gamma)(y_d^2 + \gamma^2) & (28)\\ \alpha &= q_c\mathbf{n}_c - d_c\mathbf{p}_c & (29)\\ \beta &= \frac{1}{s_x} & (30)\\ \gamma &= -\beta(k y_d + c_x) & (31)\\ y_d &= \frac{y_i - c_y}{s_y} & (32) \end{aligned}$$

4 Summary and Conclusions

In this paper, a nonlinear least squares based method for calibrating a laser stripe profiling system has been presented. A non-linear model of the system is used, which allows for radial distortion in the camera lens. A calibration target consisting of a set of planar surfaces is used. An explicit function for the error in the stripe observations off this target is developed. This is then used in a non-linear least squares algorithm. A separate linear calibration method is used to generate initial estimates of the system parameters.

A number of areas have been identified for future work. The use of centroid estimates in the preliminary calibration phase is a major source of errors. They will be replaced with corner

features, and ultimately, a linear calibration method that uses the same raw data as the non-linear phase. Another useful step will be to incorporate the relative position of the planar faces into the set of unknown system parameters, which will mean that an expensive calibration target will be unnecessary. The extension of the technique to using an arbitrary surface via ray-tracing will also be investigated.

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